

## Section 5-1 Polynomial Functions

**Learning Goal:** To understand how to classify polynomials; to understand how to graph polynomial functions and describe end behavior.

**Essential Questions:** What does the degree of a polynomial tell you about its related polynomial function?

For a polynomial function, how are factors, zeros, and x-intercepts related?

For a polynomial equation, how are factors and roots related?

### Warm Up:

- Solve equation by factoring.  $x^2 - x - 20 = 0$
- Which equation is equivalent to  $x^2 + 24x + 100 = -46$ ?
  - $(x + 12)^2 = -2$
  - $(x - 12)^2 = -2$
  - $(x - 12)^2 = 2$
  - $(x + 12)^2 = 2$
- What is the transformation of the graph of  $y = (x + 3)^2 - 2$  from its parent function  $y = x^2$ .
  - 3 units left and 2 units down
  - 3 units right and 2 units up
  - 6 units right and 2 units up
  - 2 units left and 3 units up
- What is the axis of symmetry for the graph of the quadratic equation  $y = -3x^2 - 12 + 12x$ ?
- What is the vertex of  $y = -2|x + 4| - 5$

## Vocabulary:

**Monomial** – a real number, a variable, or a product of a real number and one or more variables with whole number exponents.

Example:

**Degree of a monomial** - Is the sum of all exponents on variables

Example:

**Polynomial** – is a monomial or a sum of monomials

Example:

**Degree of a polynomial** – is the greatest degree among its monomial terms

Example:

**Polynomial Function** - a polynomial in the variable  $x$  defines a polynomial function of  $x$

Example:

**Standard form of a polynomial function**- arranges the terms by degree in descending numerical order.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where  $n$  is a nonnegative integer and  $a_n, \dots, a_0$  are real numbers

Example:  $P(x) = 4x^3 + 3x^2 + 5x - 2$

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	5	1	monomial
1	linear	$x + 4$	2	binomial
2	quadratic	$4x^2$	1	monomial
3	cubic	$4x^3 - 2x^2 + x$	3	trinomial
4	quartic	$2x^4 + 5x^2$	2	binomial
5	quintic	$-x^5 + 4x^2 + 2x + 1$	4	polynomial of 4 terms

You try:

Write each polynomial in standard form. What is the classification of each polynomial by degree? by number of terms?





1.  $3x + 9x^2 + 5$

2.  $4x - 6x^2 + x^4 + 10x^2 - 12$

3.  $3x^3 - x + 5x^4$

4.  $3 - 4x^5 + 2x^2 + 10$

Note: The degree of a polynomial function affects the shape of its graph and determines the maximum number of **turning points**, or places where the graph changes direction. It also affects the **end behavior**, or the direction of the graph to the far left and to the far right.

<b>There are 4 types of end behaviors</b>			
Up and UP	Down and Down	Down and Up	Up and Down
			

Take note

## Key Concept Polynomial Functions

$$y = 4x^4 + 6x^3 - x$$



End Behavior: Up and Up

Turning Points:  $(-1.07, -1.04)$ ,  $(-0.27, 0.17)$ , and  $(0.22, -0.15)$

The function is decreasing when  $x < -1.07$  and  $-0.27 < x < 0.22$ . The function increases when  $-1.07 < x < -0.27$  and  $x > 0.22$ .

$$y = -x^2 + 2x$$



End Behavior: Down and Down

Turning Point:  $(1, 1)$

The function is increasing when  $x < 1$  and is decreasing when  $x > 1$ .

$$y = x^3$$

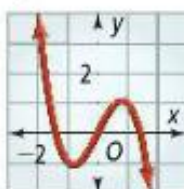


End Behavior: Down and Up

Zero turning points.

The function is increasing for all  $x$ .

$$y = -x^3 + 2x$$



End Behavior: Up and Down

Turning Points:  $(-0.82, -1.09)$  and  $(0.82, 1.09)$

The function is decreasing when  $x < -0.82$  and when  $x > 0.82$ . The function is increasing when  $-0.82 < x < 0.82$ .

### End Behavior of a Polynomial Function With Leading Term $ax^n$

	$n$ Even ( $n \neq 0$ )	$n$ Odd
a Positive	Up and Up	Down and Up
a Negative	Down and Down	Up and Down

You Try:

Consider the leading term of each polynomial function. What is the end behavior of the graph?

5.  $y = 4x^3 - 3x$

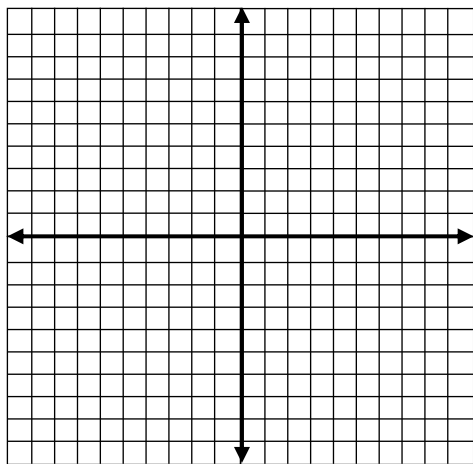
6.  $y = 2x^4 + 8x^3 - 8x^2 + 2$

7.  $y = -6x^5 - 4x^2 + 3$

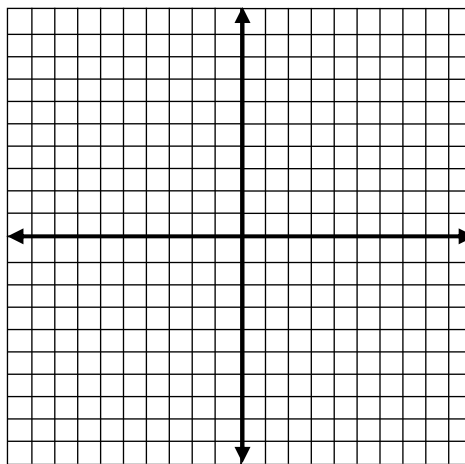
8.  $y = -8x^{10} - 13$

What is the graph of each cubic function? Describe the graph, including end behavior, turning points, and increasing/decreasing intervals.

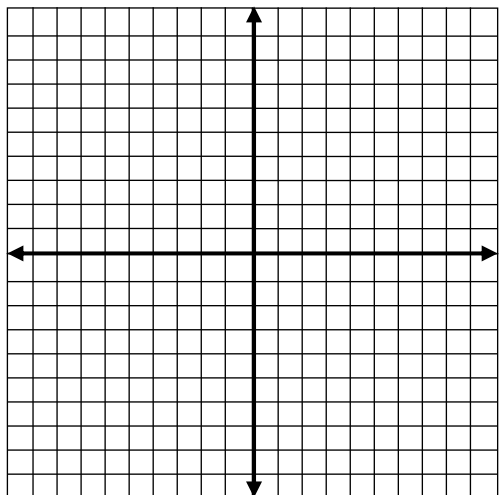
9.  $y = -\frac{1}{2}x^3$



10.  $y = x^3 + 2x^2 - x - 2$



11.  $y = \frac{1}{4}(x+3)^3 - 4$



How do you determine the degree looking at a table?

- If your input or x-values differ by a constant, you can find the difference in the output or y-values.
- If it is constant the first time it is Linear
- If it is constant the second time it is quadratic
- If it is constant the third time it is cubic
- Etc

Try some:

What is the degree of the polynomial function that generates the data shown in the table?

12.

x	y
-2	-13
-1	-4
0	-1
1	2
2	11
3	32
4	71

13.

x	y
-3	23
-2	-16
-1	-15
0	-10
1	-13
2	-12
3	29

Closure: What does the degree of a polynomial function tell you about its graph?

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