# Section 5-2 Polynomials, Linear Factors, and Zeros

**Learning Goal**:

To understand how to analyze the factored form of a polynomial; to understand how to write a polynomial function from its zeros.

**Essential Questions**: What does the degree of a polynomial tell you about its related polynomial function?

For a polynomial function, how are factors, zeros, and x-intercepts related? For a polynomial equation, how are factors and roots related?

Warm Up:

Factor:

1. 
$$x^2 + 7x + 12$$

2. 
$$x^2 + 8x - 20$$

3. 
$$x^2 - 14x + 24$$

4. 
$$3x^2 - 9x + 6$$

5. 
$$36x^2 - 84x + 49$$

6. 
$$121x^2 - 400$$

7. 
$$x^3 - 2x^2 - 15x$$

8. 
$$x^3 - x^2 - 12x$$

#### **Vocabulary:**

**Roots, Zeros, x-intercepts, and solution to the equation** - all mean the same thing.

Example: Let 
$$P(x) = -x^3 + 2x + 4$$
, and let  $x = 2$ .

Then 
$$P(2) = -(2)^3 + 2(2) + 4 = 0$$

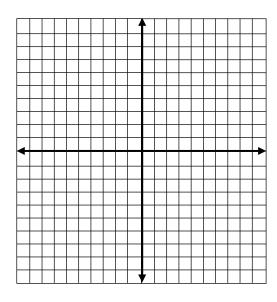
Therefore,

- x 2 is a factor of  $-x^3 + 2x + 4$
- 2 is a zero for the polynomial function P(x)
- 2 is a solution to the equation
- The point (2, 0) is an x-intercept of the graph of  $P(x) = -x^3 + 2x + 4$

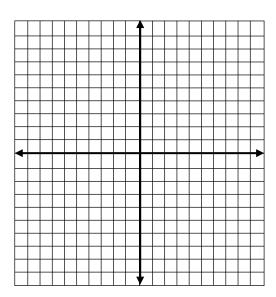
#### Finding the Zeros of a Polynomial Function

- 1. Factor the polynomial.
- 2. Use the zero-product property to find the zeros.
- 3. Find the point for the x-values between the zeros.
- 4. Using your x-values solve for y, to create an order pair.
- 5. Determine the end behavior.
- 6. Use the zeros and the additional points and the end behavior to sketch the graph .

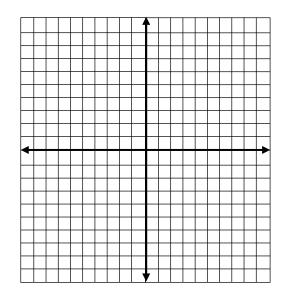
Example: What are the zeros of y = (x+2)(x-1)(x-3)? Graph the function.



1. What are the zeros of y = x(x-3)(x+5)? Graph the function.



2. What are the zeros of y = -(x-3)(x+4)(x-1)? Graph the function.



Factor Theorem -	The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the
related polynomial	function.

- 1. Identify the zeros
- 2. Write a linear factor for each zero
- 3. Multiply 2 of the binomials together
- 4. Multiply the 3<sup>rd</sup> binomial to the product of the two
- 5. Keep multiplying until all the factors are gone and you are left with a standard form polynomial.
- 6. Simplify your expression
- 7. Write as f(x) =

#### You Try:

3. What is the cubic polynomial function in standard form with zeros 1, -1, and 4?

4. What is the quartic polynomial function in standard form with zeros -2, -2, 2 and 3?

<b>Multiple Zel 05</b> - a repeated illiear factor	<b>Multiple Zeros-</b>	a repeated lin	ear facto
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<u>Multiplicity</u> – is the number of times the related linear factor is repeated in the factored form of the polynomial. (note: a graph bounces at a multiplicity point)

## **How Multiple Zeros Affect a Graph**

Example: What are the zeros of  $f(x) = x^4 - 2x^3 - 8x^2$ . How does the graph behave at these zeros?

### You Try:

5. What are the zeros of  $f(x) = x^3 - 5x^2 + 3x + 9$ ? What are the multiplicities? How does the graph behave at these zeros?

The graph of a polynomial function can have several turning points, the function can have a relative maximum and a relative minimum.

**<u>Relative Maximum</u>** – is the value of the function at an up to down turning point

**<u>Relative Minimum</u>** – is the value of the function at a down to up turning point

6. What are the relative maximum and minimum of  $f(x) = x^3 - 9x$ ?

Closure:	What do the zeros of a polynomial function tell you about the function's graph?			

Assignment: section 5.2 # 7,10,11,12,14,15,16,20,21,27,29,31,32,35,44 (15 problems)