Name: _____   Date: _____   # _____

Section 6-7 Inverse Relations and Functions

Learning Goal: To understand how to find the inverse of a relation or function.

Essential Questions: How can you simplify the nth root of an expression that contains an nth root as a factor?
When you square each side of an equation, is the resulting equation equivalent to the original?
How are function and its inverse function related?

Warm Up:

1.   Let $f(x)=\sqrt{x}-4$ and $g(x)=x+6$. What are $f+g$ and $f-g$? What are their domains?

     a) $f+g$ b) $f-g$

2.   Let $f(x)=x^2-16$ and $g(x)=x-4$. What are $f\cdot g$ and $\dfrac{f}{g}$? What are their domains?

     a) $f\cdot g$ b) $\dfrac{f}{g}$

3.   Let $f(x)=x-2$ and $g(x)=x^3$. What is $(f\circ g)(2)$?

Vocabulary:

Relation: a set of ordered pairs (x, y)

Function: a function is a relation in which each element of the domain corresponds with exactly one element in the range.

Domain: x – values

Range: y-values

Inverse relation: is a set of ordered pairs (y, x)

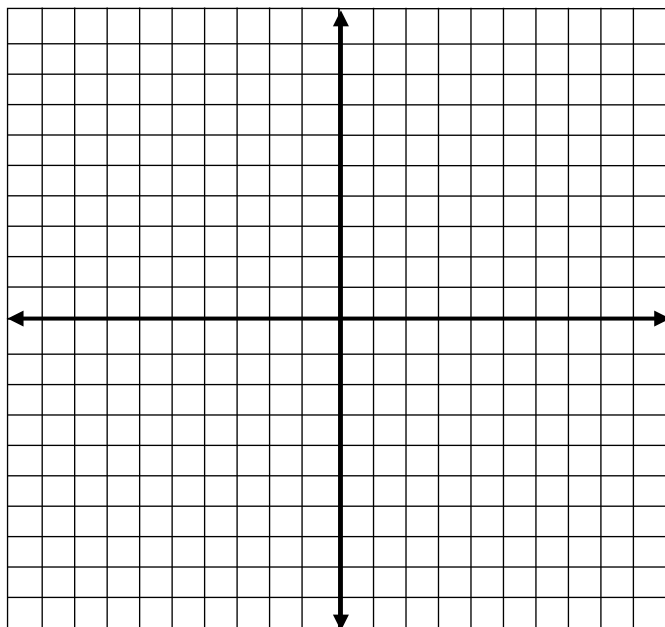*Note:* if both a relation and its inverse happen to be functions, they are **inverse functions**.

You Try:

1. What is the inverse of relation s?

    Relation s

x	y
0	-1
2	0
3	2
4	3

2. What are the graphs of s (mark with a dot) and its inverse(mark with an x).

3. What is the inverse of relation v?

Relation v

x	y
-2	2
-1	0
0	3
1	0

4. What are the graphs of v (mark with a dot) and its inverse(mark with an x).
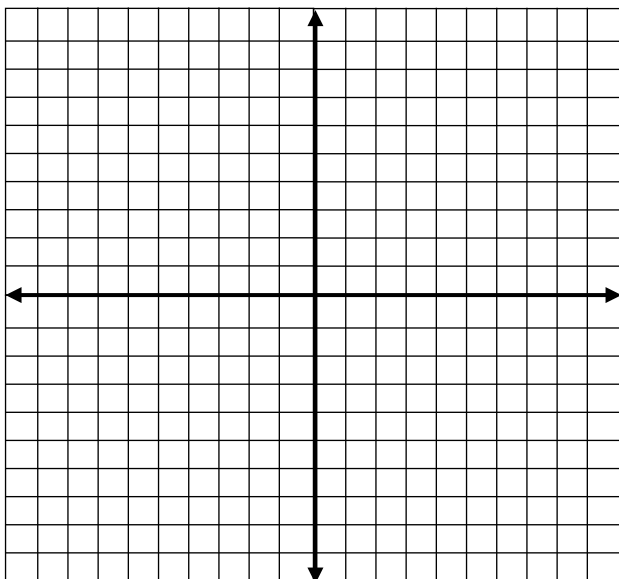
Note: The graphs of a relation and its inverse are the reflections of each other in the line y = x. If you describe a relation of function by an equation in x and y, you can switch x and y to get an equation of the inverse.
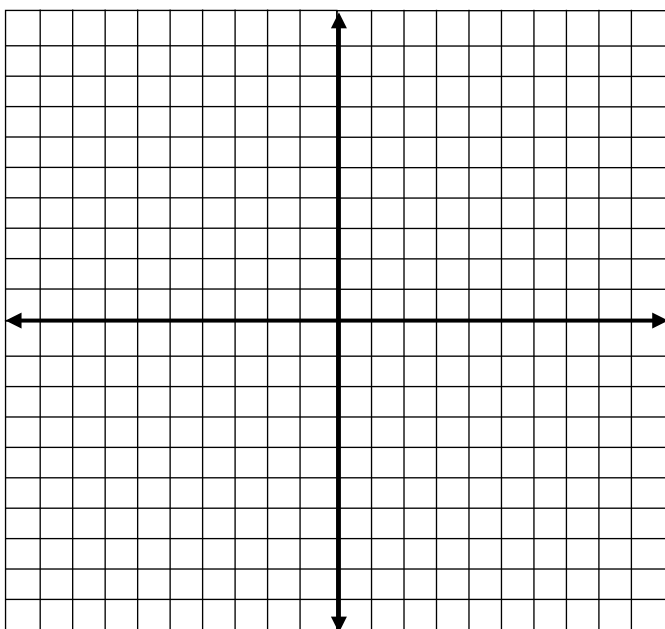
5. What is the inverse of the relation described by $y = x^2 - 1$?

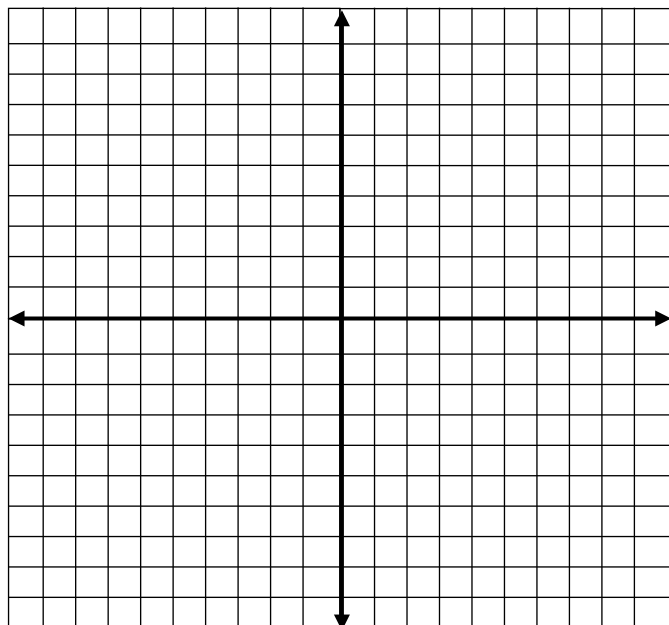6. What is the inverse of the relation described by $y = 2x + 8$?

7. What is the inverse of the relation described by $y = 5x^2 + 2$?

Graphing a relation and its inverse.

8. What are the graphs of $y = x^2 - 1$ and its inverse, $y = \pm\sqrt{x+1}$?

9. What are the graphs of $y = 2x + 8$ and its inverse?

Inverse of a function f – denoted by f^{-1}. Read " the inverse of f" or "f inverse".

The notations f(x) is used for functions but the relation f^{-1} may not be a function.

Finding and Inverse function.

1. What are the domain and range of a function?
2. Find f^{-1}, the inverse of f.
3. What are the domain and range of f^{-1}?
4. Is f^{-1} a function? Yes, if for every x in the domain, there is only one y in the range.
   No, if 1 domain has two different ranges.

Example:

Consider the function $f(x) = \sqrt{x-2}$.

    a) What are the domain and range of f?

    b) What is f^{-1}, the inverse of f?

    c) What are the domain and range of f^{-1}?

    d) Is f^{-1} a function? Explain.

You Try:

10. Consider the function $g(x) = 6 - 4x$.

      a) What are the domain and range of g?
      b) What is g^{-1}, the inverse of g?
      c) What are the domain and range of g^{-1}?
      d) Is g^{-1} a function? Explain.

One-to-one function: is a function for which each y-value in the range corresponds to exactly one x-value in the domain. A one-to-one function f has an inverse f^{-1} that is also a function.

<u>Composition of Inverse Functions:</u>   If f and f^{-1} are inverse functions, then

     $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1}, respectively

Your try:

11.   For $f(x) = \dfrac{1}{x-1}$, what is each of the following?

   a) $f^{-1}(x)$

   b) $\left(f \circ f^{-1}\right)(1)$

   c) $\left(f^{-1} \circ f\right)(1)$

12. For $g(x) = \dfrac{4}{x+2}$, what is each of the following?

a) $g^{-1}(x)$

b) $\left(g \circ g^{-1}\right)(0)$

c) $\left(g^{-1} \circ g\right)(0)$

Closure: How can you tell from the graph of a function whether its inverse is a function?

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Assignment: section 6.7 # 8,11,13,15,21,23,26,27,31,35,38,42,44,52,55   (15 problems)