

Name: \_\_\_\_\_

Date: \_\_\_\_\_ # \_\_\_\_\_

## Section 8-1 Inverse Variations

**Learning Goal:** To understand how to recognize and view inverse variations: to use joint and other variations

**Essential Questions:** Are two quantities inversely proportional if an increase in one corresponds to a decrease in the other?

What kinds of asymptotes are possible for a rational function?

Are a rational expression and its simplified form equivalent?

### Warm Up:

1. For each direct variation ( $y=kx$ ), find the constant of variation. Then find the value of  $y$  when  $x = -3$ .

a)  $y = 4$  and  $x = 3$

b)  $y = -5$  and  $x = \frac{3}{2}$

2. Factor each expression.

a)  $x^2 + x - 6$

b)  $4x^2 + 17x + 15$

c)  $9x^2 - 25$

d)  $x^2 - 12x + 36$

e)  $3x^2 + 10x + 8$

f)  $x^2 - 5x + 6$

**Vocabulary:**

**Recall:**

**Direct variation** has the form  $y = kx$ , where  $k \neq 0$  “y varies directly with x”

**Inverse variation** – Two quantities vary inversely, as one quantity increases, the other decreases proportionally.

of the form:  $xy = k$ ,  $y = \frac{k}{x}$ , or  $x = \frac{k}{y}$  where  $k \neq 0$

**Identify Direct and Inverse Variation**

You Try:

Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for the direct and inverse variations.

Given Equation	Rewritten Equation	Type of Variation
1. $\frac{y}{5} = x$		
2. $y = x + 2$		
3. $xy = 4$		
4. $xy = 4.8$		
5. $x = \frac{y}{1.5}$		
6. $4y = x^2$		

Is the relationship between the variables a direct variation, an inverse variation, or neither? Write function models for the direct and inverse variations.

7.

<b>x</b>	<b>y</b>
2	15
4	7.5
10	3
15	2

8.

<b>x</b>	<b>y</b>
2	10
4	8
10	3
15	1.5

9.

<b>x</b>	<b>y</b>
0.2	8
0.5	20
1.0	40
1.5	60

10.

<b>x</b>	<b>y</b>
0.2	40
0.5	16
1.0	8.0
2.0	4.0

11.

<b>x</b>	<b>y</b>
0.5	40
1.2	12
2	10
2.5	6

12.

<b>x</b>	<b>y</b>
1	52
2	34
5	4
6	2

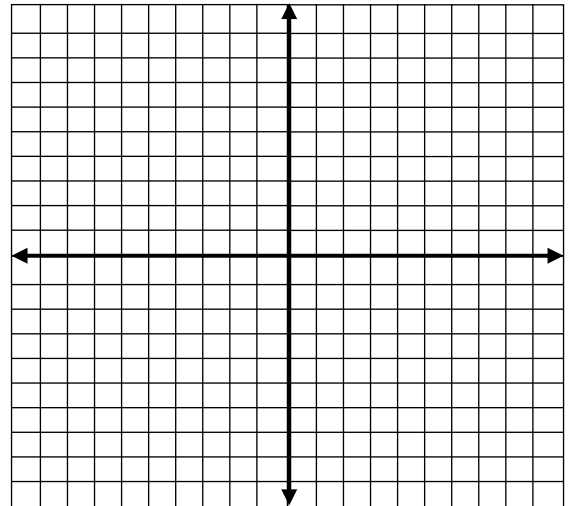
**Steps for Determining an Inverse Variation:**

1. Write the general equation for the inverse variation  $y = \frac{k}{x}$
2. Substitute the x and y values into the equation  $y = \frac{k}{x}$
3. Solve for k.  $k=xy$
4. Write the inverse variation equation.

Example: Suppose x and y vary inversely, and x =4 when y =12.

a) What function models the inverse variation?

b) What does the graph of this function look like?



c) What is y when x = 10?

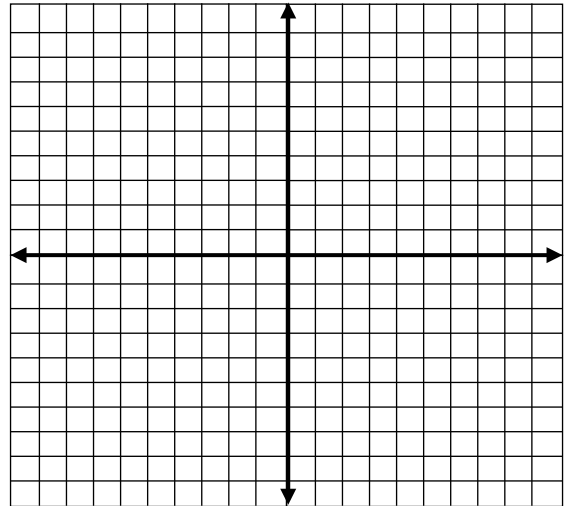
You Try:

13. Suppose  $x$  and  $y$  vary inversely, and  $x = -8$  when  $y = -7$ .

a) What is the function that models the inverse variation?

b) What does the graph of this function look like?

c) What is  $y$  when  $x = 2$ ?

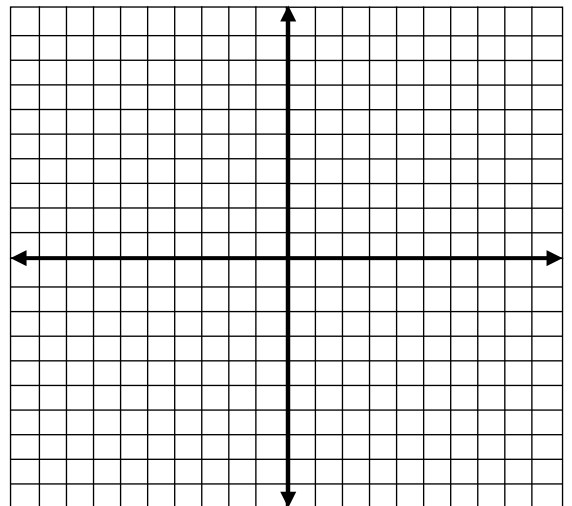


14. Suppose  $x$  and  $y$  vary inversely, and  $x = 2$  when  $y = 8$ .

a) What is the function that models the inverse variation?

b) What is the graph of this function?

d) What is  $y$  when  $x = 4$ ?



## Modeling an Inverse Variation

Your math class has decided to pick up litter each weekend in a local park. Each week there is approximately the same amount of litter. The table shows the number of students who worked each of the first four weeks of the project and the time needed for the pickup.

Park Cleanup Project				
Number of students (n)	3	5	12	17
Time in minutes (t)	85	51	21	15

- a) What function models the data?

**Step 1:** Investigate the data. The more students who help, the less time the cleanup takes. An inverse variation seems appropriate. If this is an inverse variation, then  $nt = k$ . From the table,  $nt$  (or  $L1 \cdot L2$ ) is almost always 255.

**Step 2:** Determine the model,  $nt = 255$

- b) How many students should there be to complete the project in at most 30 minutes each week?

Use the model:  $nt = 255$

Plug in 30 for the  $t$  (time)

Solve for  $n$ .

There should be at least \_\_\_\_\_ students to do the job in at most 30 minutes.

15. After a major storm, your math class volunteers to remove debris from yards. The table shows the time  $t$  in minutes that it takes a group of  $n$  student to remove the debris from an average sized yard.

Average Sized Yard clean up				
Number of students (n)	1	3	5	14
Time in minutes (t)	225	75	45	16

- a) What function models the time needed to clear the debris from an average-sized yard relative to the number of students who do the work?

- b) How many students should there be to clear the debris from an average-sized yard in at most 25 minutes?

16. Your employer decides to hire extra help to deliver newspapers during the holidays. The table shows the number of employees who delivered papers each week and the time needed for the delivery.

<b>Newspaper delivery help</b>				
Number of students (n)	2	4	19	20
Time in minutes (t)	100	50	20	10

a) What function models this problem?

b) How many employees should there be to deliver newspapers in at most 40 minutes each week?

17. The speed of the current in a whirlpool varies inversely with the distance from the whirlpool located off the coast of Norway. At a distance of 3 kilometers (3000 meters) from the center, the speed of the current is about 0.1 meter per second. Describe the change in speed of the current as you move closer to the whirlpool's center.

Complete the table:

Distance from center (meters), d	Speed (meters per second), s
2000	
1500	
500	
250	
50	

18. The volume of a gas in a container varies inversely with the amount of pressure. A gas has volume  $75 \text{ in}^3$  at a pressure of  $25 \text{ lb/in}^2$ . Write a model relating volume and pressure.

Sometimes a formula might relate 3 or more variables.

**Combined Variation:** When one quantity varies with respect to two or more quantities

**Joint Variation:** when one quantity varied directly with two or more quantities

Ex: volume of cone varies jointly with the area of the base and the height of the cone.  $V=kBh$

Combined Variations	
Combined Variation	Equation Form
z varies jointly with x and y	$z = kxy$
z varies jointly with x and y and inversely with w	$z = \frac{kxy}{w}$
z varies directly with x and inversely with the product of wy	$z = \frac{kx}{wy}$

You Try:

Write an equation for the given relationship.

Relationship	Equation
19. y varies directly with x	
20. y varies inversely with x	
21. z varies jointly with x and y	
22. y varies inversely with the square of x	
23. z varies directly with y and inversely with x	
24. y varies directly with x and inversely with $z^2$	
25. y varies inversely with $x^3$	
26. y varies directly with $x^2$ and inversely with z	
27. z varies jointly with $x^2$ and y	
28. y varies inversely with x and z	



## Using Combined Variation

The number of bags of grass seed  $n$  needed to reseed a yard varies directly with the area to be seeded and inversely with the weight  $w$  of a bag of seed. If it takes two 3-lb bags to seed an area of  $3600 \text{ ft}^2$ , how many 3-lb bags will seed  $9000 \text{ ft}^2$ ?

You Try:

29. The number of bags of mulch you need to cover a planting area varies jointly with the area to be mulched  $a$  in square feet and the depth of the mulch  $d$  in feet. If you need 10 bags to mulch  $120 \text{ ft}^2$  to a depth of 3 in., how many bags do you need to mulch  $200 \text{ ft}^2$  to a depth of 4 in.?

30. The volume of a cone varies jointly with its height and the square of its base radius. A cone has a base radius of 4 ft, height 6 ft, and volume  $100.48 \text{ ft}^3$ . What is the volume of a cone with height of 3 ft and a base radius of 3 ft?

## Applying Combined Variation

Gravitational potential energy, PE is a measure of energy. PE varies directly with an object's mass  $m$  and its height  $h$  in meters about the ground. Physicists use  $g$  to represent the constant of variation, which is gravity.

A skateboarder has a mass of 58 kg and a potential energy of 2273.6 joules. What is the gravitational potential energy of a 65-kg skateboarder on a halfpipe that is 4 meters high?

You Try:

31. How much potential energy would a 41-kg diver have standing on a 10-m diving platform?

32. The volume of gas varies directly with its temperature and inversely with pressure. Volume is  $100 \text{ m}^3$  when the temperature is 150 K and the pressure is  $15 \text{ lb/cm}^2$ . What is the volume when the temperature is 250 K and the pressure is  $20 \text{ lb/cm}^2$ ?

Closure: How can you tell whether two sets of data show direct variation or inverse variation?

---

---

---

Assignment: section 8.1 # 6,7,10\*,12,13,17\*,19,20,23,26,27,29,34,38 (14 problems) \*do not graph