Key

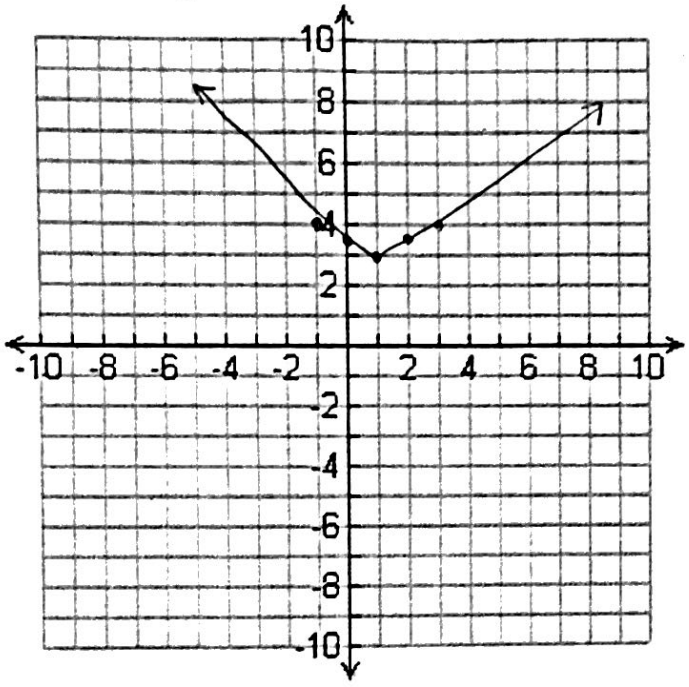
Dynamics Final Rev.

Worked out solutions to these problems will be posted ~~to eBackpack after class on Friday, June 12th.~~ There is a formula sheet attached to the back for easy reference that will also be provided on the final. There is a list of practice problems from the iBook at the end of this packet for additional practice.

1. What is the vertex of the equation $y=-2|x+5|-4$? V(-5, -4)

$y = a|x-h|+k$

2. Graph $y=\frac{1}{2}|x-1|+3$

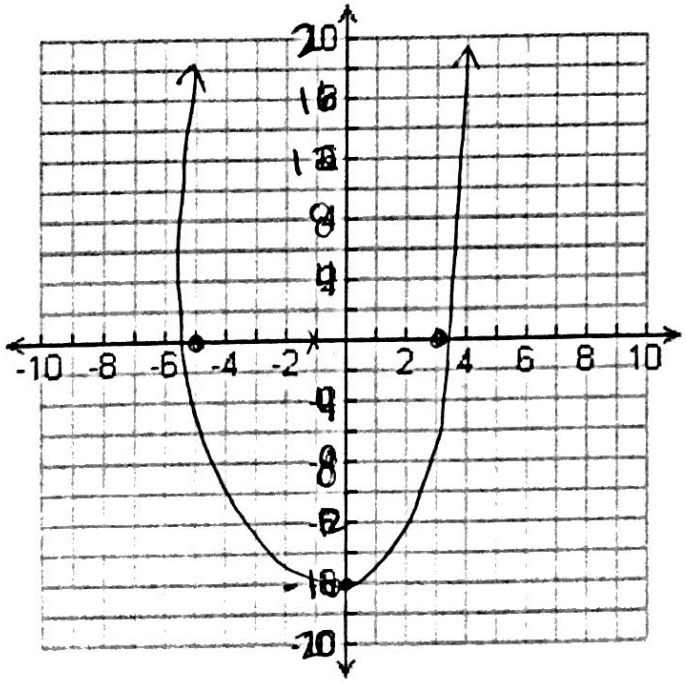


V(1, 3)

x	y
-1	4
0	3.5
V 1	3
2	3.5
3	4

3. Graph $y=(x-3)(x+5)$

$y = (x-3)(x+5)$



$x-3=0 \quad x+5=0$

$x=3 \quad x=-5$

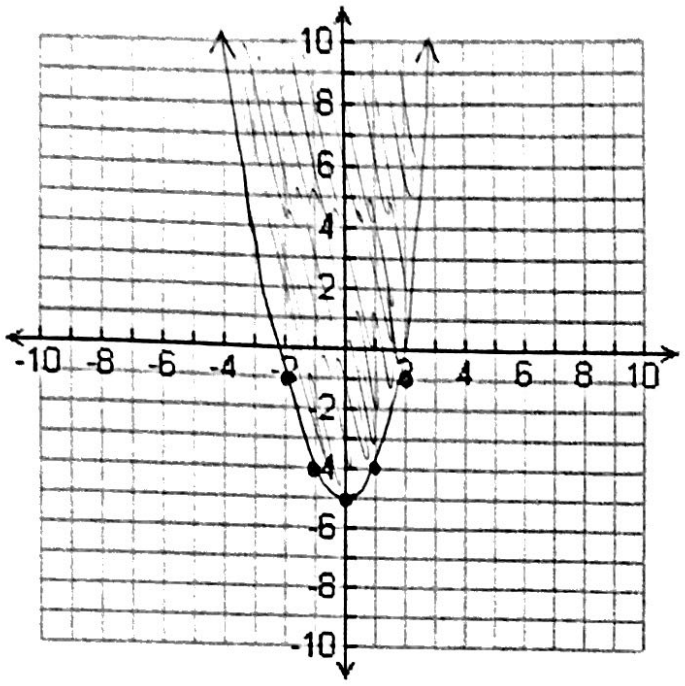
(-1, -16)

$y = (-1-3)(-1+5)$

$(-4)(4)$

-16

4. Graph $y \geq x^2 - 5$



V(0, -5)

-2	-1
-1	-4
V 0	-5
1	-4
2	-1

test pt (0,0)

$y \geq x^2 - 5$

$0 \geq 0 - 5$

$0 \geq -5$

✓

5. Write an example of a quadratic trinomial in **standard form**. $5x^2 - 4x + 3$

6. Solve the equation using square roots: $2x^2 - 1 = 39$ (Don't forget the ± !)

$+1 \quad +1$

$\frac{2x^2}{2} = \frac{40}{2}$

$\sqrt{x^2} = \sqrt{20}$

$x = \pm\sqrt{4}\sqrt{5}$

$\boxed{x = \pm 2\sqrt{5}}$

7. Factor completely.

a. $3x^2 - 11x - 4$

$(3x+1)(x-4)$

b. $x^2 - 7x + 6$

$(x-6)(x-1)$

7 (continued). Factor completely.

c. x^3+3x^2-2x-6 (Factor by grouping!)

$x^2(x+3)-2(x+3)$

$(x^2-2)(x+3)$

d. x^4+x^2-20

$(x^2+5)(x^2-4)$

$(x^2+5)(x-2)(x+2)$

8. Solve: $x^2+6x=0$

$x(x+6)=0$

$x=0 \quad x=-6$

9. What is the vertex of $y=\frac{1}{2}(x-3)^2+5$? *Recall that there is something special about h!*

$V(3,5)$

10. What is the vertex of $y=2x^2+16x+39$? *There is a formula we use to find x, which is* $x=\frac{-b}{2a}$

$a=2$
$b=16$
$c=39$

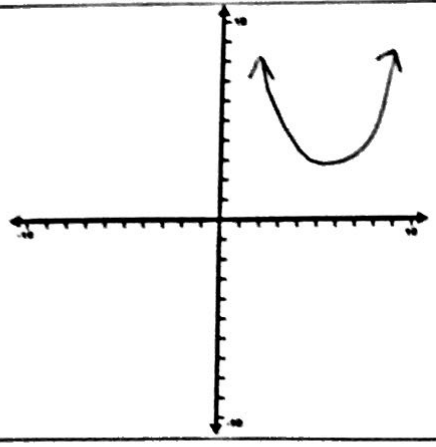
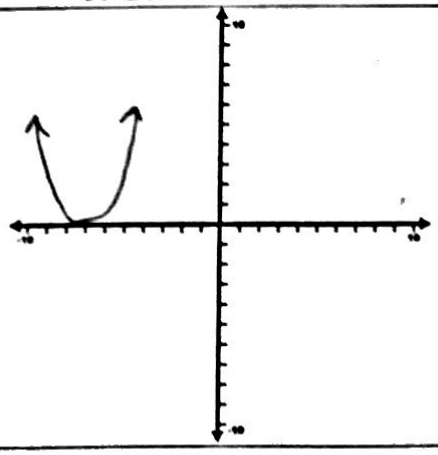
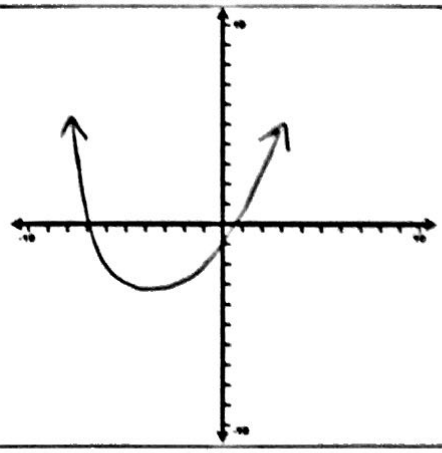
$x=\frac{-16}{2(2)}=\frac{-16}{4}=-4$

$y=2(-4)^2+16(-4)+39$

$y=7$

$V(-4,7)$

11. Draw an example of a quadratic with the following types of solutions. (Remember that the graph of a quadratic is a parabola!)

ZERO REAL SOLUTION	ONE REAL SOLUTION	TWO REAL SOLUTIONS
		

12. Simplify completely.

a. $\frac{7}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{7\sqrt{2}}{2}}$

b. $\frac{20}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5}$

$\boxed{4\sqrt{5}}$

c. $\sqrt{-7}$ $\sqrt{-1}\sqrt{7}$

$\boxed{i\sqrt{7}}$

d. $\sqrt{-12}$ $\sqrt{-1}\sqrt{4}\sqrt{3}$

$\boxed{2i\sqrt{3}}$

e. $\sqrt{21}\cdot\sqrt{-7}$

$\sqrt{-147}$

$\sqrt{-1}\sqrt{49}\sqrt{3}$

$\boxed{7i\sqrt{3}}$

f. $\sqrt{25x^8y^4}$

$\boxed{5x^4y^2}$

13. Solve $2x^2-3x-10=0$ using the Quadratic Formula. (Find the Quadratic Formula on the reference sheet!)

$a=2$
$b=-3$
$c=-10$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad x = \frac{3\pm\sqrt{(-3)^2-4(2)(-10)}}{2(2)}$

$\boxed{x = \frac{3\pm\sqrt{89}}{4}}$

14. Write the expression as a complex number in standard form:

a. $(6+9i)+(1-2i)$

$\boxed{7+7i}$

b. $(8-9i)(2+3i)$

$16+24i-18i-27i^2$ $\quad (-1)$

$16+6i+28$

$\boxed{44+6i}$

15. Factor completely: $5x^3 - 5x^2 + 10x$

$5x(x^2 - x + 2)$

$5x(x-2)(x+1)$

16. Simplify using the properties of exponents. Do not leave negative exponents in your final answer!

a. $(6x^3y^4)^2$

$36x^6y^8$

b. $\left(\frac{4a^4b}{5a^6b^{-2}}\right)^3$

$\frac{4^3a^{12}b^3}{5^3a^{18}b^{-6}} = \frac{64a^{12}b^3b^6}{125a^{18}}$

$\frac{64b^9}{125a^6}$

c. $(2x^{-2})^{-3}$ $2^{-3}x^6$

$\frac{x^6}{8}$

17. Simplify.

a. $(3x-7)+(2x^2-6x+11)$

$2x^2 - 3x + 4$

b. $(4x^2-7)-(2x-13)$

$4x^2 - 7 - 2x + 13$

$4x^2 - 2x + 6$

c. $(2x-9)(x^2-x+5)$

$2x^3 - 2x^2 + 10x - 9x^2 + 9x - 45$

$2x^3 - 11x^2 + 19x - 45$

d. $(3x+4)^2$

$(3x+4)(3x+4)$

$9x^2 + 12x + 12x + 16$

$9x^2 + 24x + 16$

18. Divide $(5x^2+4x-3)\div(x+3)$ using synthetic division.

$$\begin{array}{r|rrr} -3 & 5 & 4 & -3 \\ & \downarrow & -15 & 33 \\ \hline & 5 & -11 & 30 \end{array}$$

$5x^2 - 11 \text{ R } \frac{30}{x+3}$

a c

19. List all the possible rational roots of the following polynomial: $y = -3x^4 - 7x + 15$

$\frac{c}{a}$: $\frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 3}$ $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}$

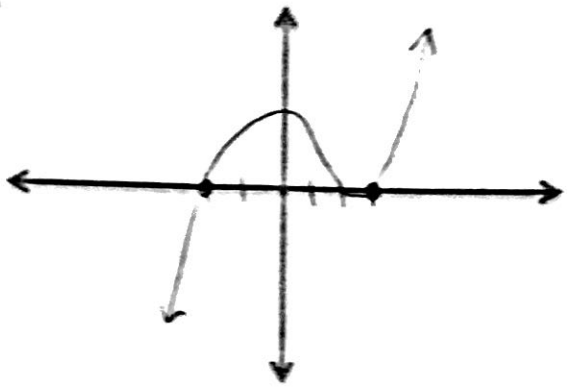
20. Sketch a graph of $y = (x+2)(x-3)^2$

$x+2=0$ $x-3=0$

$x=-2$ $x=3$

mult of 2

$x^3 \downarrow \uparrow$



21. Classify each of the following by degree.

a. $y = 3x^4$ b. $y = -5x^3 - x^2 + 7$ c. $y = -4x^2 - 1$ d. $y = 4x^5 + 9$

4= Quartic 3= Cubic 2= Quadratic 5= Quintic

22. Write in exponential notation: $\left(\sqrt[3]{5x}\right)^7$

$(5x)^{7/3} = 5^{7/3} x^{7/3}$

23. Write $(36x^6)^{\frac{3}{2}}$ in radical notation and simplify if possible.

$\left(\sqrt{36x^6}\right)^3$

$(6x^3)^3$

$\boxed{216x^9}$

24. Solve.

a. $\left(\sqrt[3]{40x+5}\right)^3 = (5)^3$

$40x+5 = 125$

$-5 \quad -5$

$\frac{40x}{40} = \frac{120}{40}$

$\boxed{x=3}$

Check:

$\sqrt[3]{40\cdot 3+5} = 5$

$\sqrt[3]{125} = 5$

$5 = 5$ ✓

b. $(x-1)^{\frac{3}{2}} + 4 = 31$

$-4 \quad -4$

$\left((x-1)^{3/2}\right)^{2/3} = (27)^{2/3}$

$x-1 = 9$

$+1 \quad +1$

$\boxed{x=10}$

Check

$(10-1)^{3/2} + 4 = 31$

$9^{3/2} + 4 = 31$

$27 + 4 = 31$

$31 = 31$ ✓

25. Find the inverse of $f(x) = 2x-1$.

$x = 2y - 1$

$+1 \quad +1$

$\frac{2y}{2} = \frac{x+1}{2}$

$\boxed{y = \frac{x}{2} + \frac{1}{2}}$

26. Given $f(x) = 4x+3$ and $g(x) = x^2-6$

a. Find $f(g(2))$

$f(g(2))$

$f(-2)$

$\boxed{f(g(2)) = -5}$

$g(2) = 2^2 - 6$

$4-6$

$g(2) = -2$

$f(-2) = 4(-2)+3$

$-8+3$

$f(-2) = -5$

b. $g(x) - f(x)$

$(x^2-6) - (4x+3)$

$x^2 - 6 - 4x - 3$

$\boxed{x^2 - 4x - 9}$

27. Simplify: $(3-\sqrt{5})(3+\sqrt{5})$

$9 + 3\sqrt{5} - 3\sqrt{5} - \sqrt{25}$

$9 - 5$

$\boxed{4}$

28. Simplify: $\sqrt{12}+\sqrt{8}-\sqrt{72}$

$\sqrt{4}\sqrt{3}+\sqrt{4}\sqrt{2}-\sqrt{36}\sqrt{2}$

$2\sqrt{3}+2\sqrt{2}-6\sqrt{2}$

$\boxed{2\sqrt{3}-4\sqrt{2}}$

squares

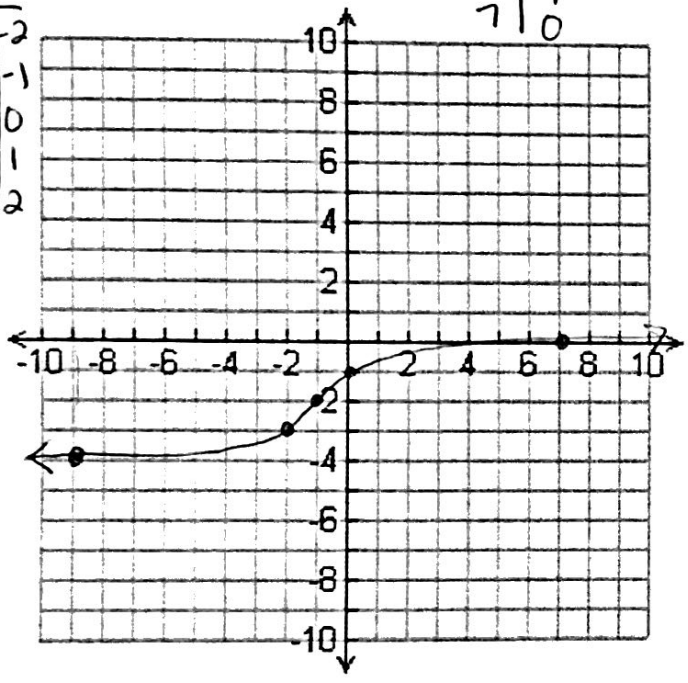
0	0
1	1
4	2
9	3

29. Graph $y=\sqrt[3]{x+1}-2$

cubes

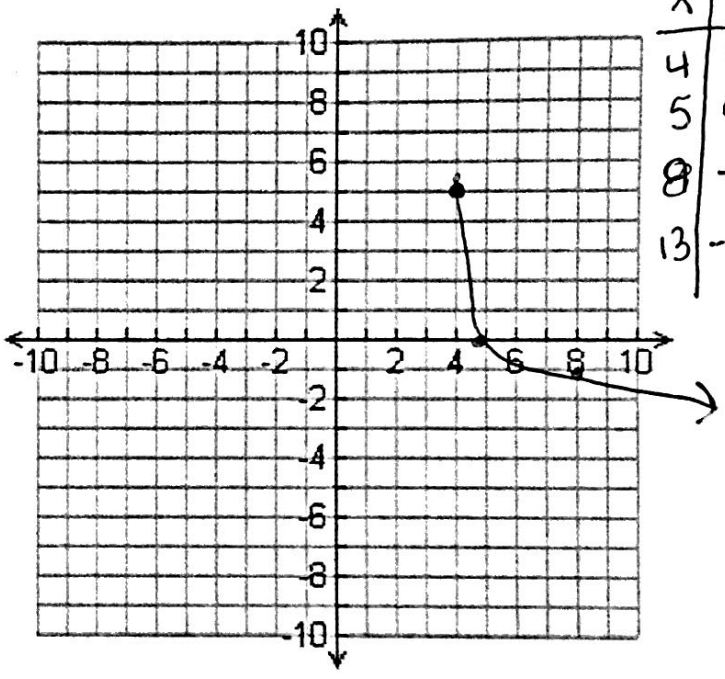
-8	-2
-1	-1
0	0
1	1
8	2

-9	-4
-2	-3
-1	-2
0	-1
7	0



30. Graph $y=-\sqrt{x-4}+1$

x	y
4	1
5	0
8	-1
13	-2



31. Convert to logarithmic notation: $3^4=81$

$\log_3 81 = 4$

32. Evaluate $\log_2 64 = x$

$2^x = 64$

$2^x = 2^6$

$\boxed{x=6}$

33. Condense $5\log_4 x+\log_4 y-\log_4 z$

$\log_4 \frac{x^5 y}{z}$

34. Expand $\log_3 \frac{7\sqrt{x}}{y^8}$ $= \log_3 \frac{7x^{1/2}}{y^8}$

$\log_3 7 + \frac{1}{2}\log x - 8\log y$

35. State whether $f(x)$ represents exponential growth or exponential decay or neither.

a. $f(x) = -3(4)^x$

growth

$b > 1$

b. $f(x) = \frac{1}{2}(4)^{-x}$

$f(x) = \frac{1}{2}\left(\frac{1}{4}\right)^x$

decay

$0 < b < 1$

c. $f(x) = \left(\frac{4}{3}\right)^x$

growth

$b > 1$

d. $f(x) = 2\left(\frac{1}{5}\right)^x$

decay

$0 < b < 1$

36. Solve for x: $16 = 2^{x-5}$

$2^4 = 2^{x-5}$

$4 = x - 5$

$+5 \quad +5$

$x = 9$

or

$16 = 2^{x-5}$

$\frac{\log 16}{\log 2} = \frac{x-5\log 2}{\log 2}$

$4 = x - 5$

$+5 \quad +5$

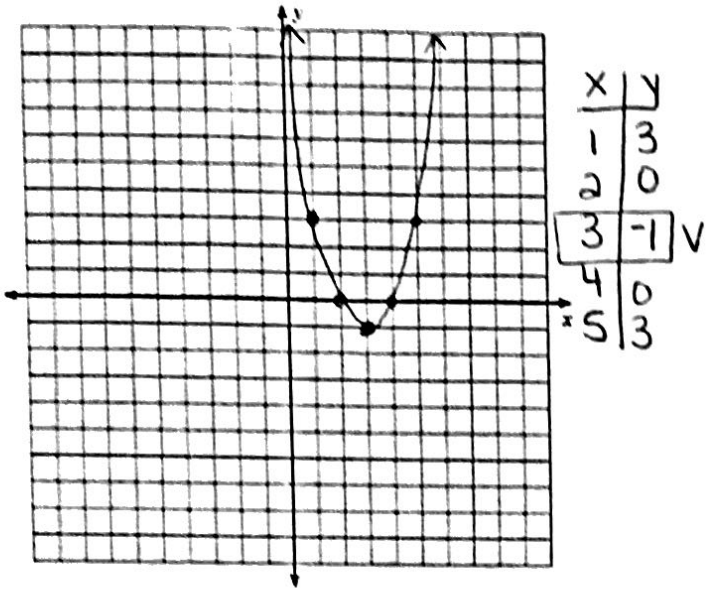
$x = 9$

Graph the following quadratic functions. Label all coordinates.

37. $f(x)=x^2-6x+8$ $x=\frac{-b}{2a}=\frac{6}{2(1)}=\frac{6}{2}=3$

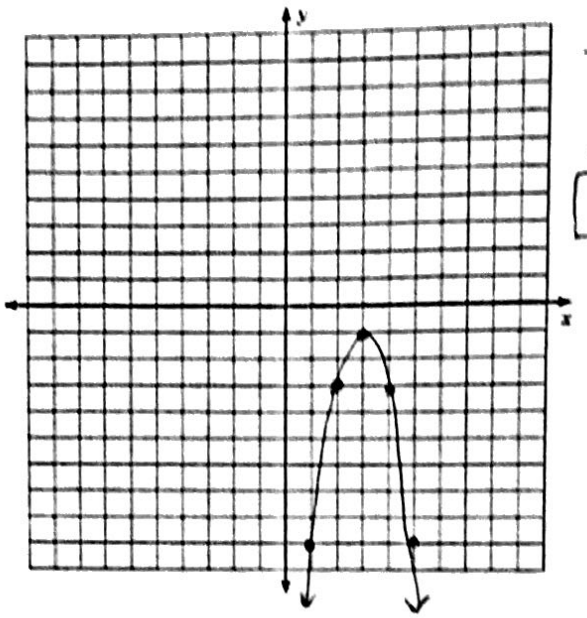
$V(3,-1)$ $y=3^2-6(3)+8$

$y=-1$



x	y
1	3
2	0
3	-1 V
4	0
5	3

38. $g(x)=-2(x-3)^2-1$ $V(3,-1)$



1	-9
2	-3
3	-1 V
4	-3
5	-9

Using the graphs above, write two similarities and two differences between $f(x)$ and $g(x)$.

Similarities

* 1. Vertex both (3,-1)
2. parabolas
* 3. same axis of symmetry.

Differences

1. opens up / down
2. normal / narrow stretched

39. List the following formulas used for exponential word problems:

a. Growth: $y = ab^x$

b. Decay: $y = ab^x$

c. Compound Interest: $A = P(1 + \frac{r}{n})^{nt}$

d. Compound Continuously: $A = Pe^{rt}$

40. A photocopier, which originally costs $500,000, depreciates exponential by 10% each year.

e. Write a model to represent this situation. $b = (1 - .10)$

$y = ab^x$ $y = 500{,}000(.90)^x$

f. What will the photocopier's value be worth in 5 years?

$y = 500{,}000(.90)^5$

$y = 295{,}245$

$295,245

41. Mark invests $500 in a savings plan that pays interest, which is compounded monthly. If the interest rate is 6%, how much will Mark have after 10 years?

$A = ?$
$P = 500$
$r = 6\% = .06$
$n = 12$ (monthly)
$t = 10$ years

$A = 500(1 + \frac{.06}{12})^{12 \cdot 10}$

$A = 909.70$

$909.70

42. The population of Littleton is currently 23,000. Assume that Littleton's exponential growth rate is 2% per year.

a. Write a model to represent this situation. $b = (1 + .02)$

$y = ab^x$ $y = 23000(1.02)^x$

b. Predict the population in 7 years.

$y = 23000(1.02)^7$

$y = 26419.77$

about 26,420 people

43. Suppose $5000 is put into an account that pays 0.9% interest compounded continuously. How much will be in the account after 3 years?

$A =$
$P = 5000$
$r = .9\% = .009$
$t = 3$ yrs

$A = Pe^{rt}$ $A = 5000e^{.009(3)}$

$A = 5136.84$

$5136.84