

## Section 7-1 Exploring Exponential Models

**Learning Goal:** To understand how to model exponential growth and decay.

**Essential Questions:** How do you model quantity that changes regularly over time by the same percentage?

How are exponents and logarithms related?

How are exponential functions and logarithmic functions related?

**Warm Up:**

Evaluate each expression for  $x = -2, 0,$  and  $2$ .

	1. $10^{x+1}$	2. $\left(\frac{3}{2}\right)^x$	3. $-5^{x-2}$
$x = -2$			
$x = 0$			
$x = 2$			

4. In advertising, the decay factor describes how an advertisement loses its effectiveness over time. In math, would you expect a decay factor to increase or decrease the value of  $y$  as  $x$  increases?

5. There are different kinds of growth patterns. Patterns that increase by a constant rate are linear. Patterns that grow exponentially increase by an ever-increasing rate. If your allowance doubles each week, does that represent linear growth or exponential growth?

6. The word asymptote comes from a Greek word meaning "not falling together." When looking at the end behavior of a function, do you expect the graph to intersect its asymptote?

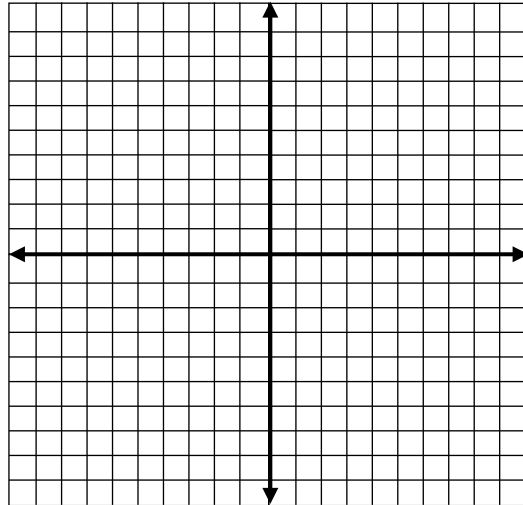
## Vocabulary:

**Exponential function** – is a function of the form  $y = ab^x$ ,  $a \neq 0$ , with  $b > 0$ , and  $b \neq 1$ . The base  $b$  is a constant. The exponent  $x$  is the independent variable with the domain the set of real numbers.

## Graphing an Exponential Function

What is the graph of  $y = 2^x$ ?

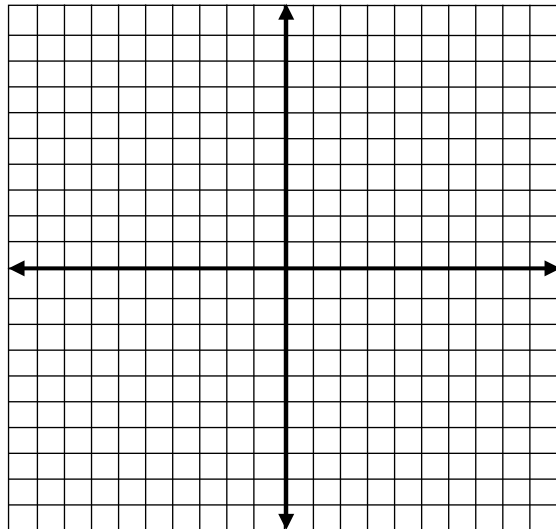
x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	



You Try:

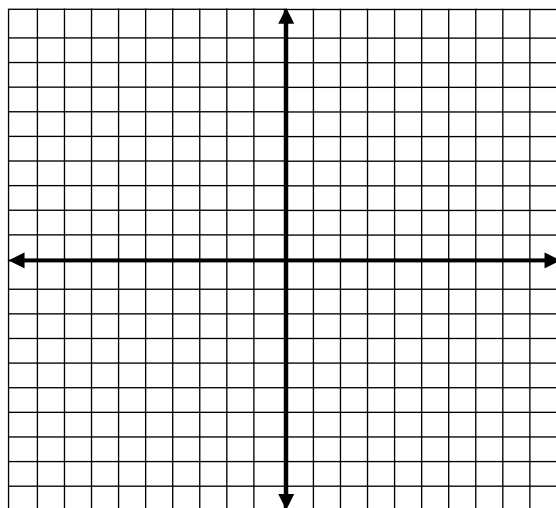
1. What is the graph of  $y = 4^x$ ?

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	



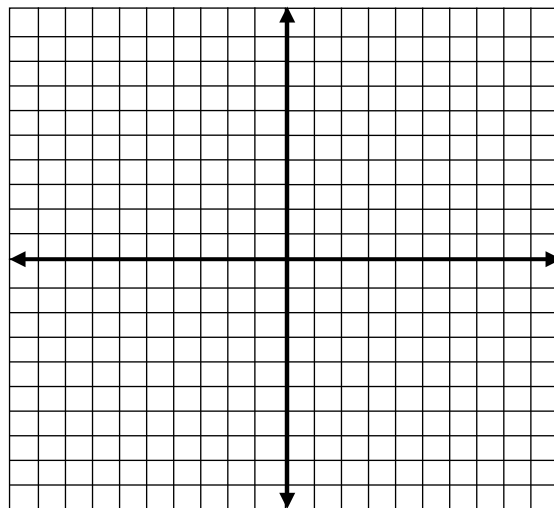
2. What is the graph of  $y = \left(\frac{1}{3}\right)^x$ ?

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	



3. What is the graph of  $y = 2(3)^x$ ?

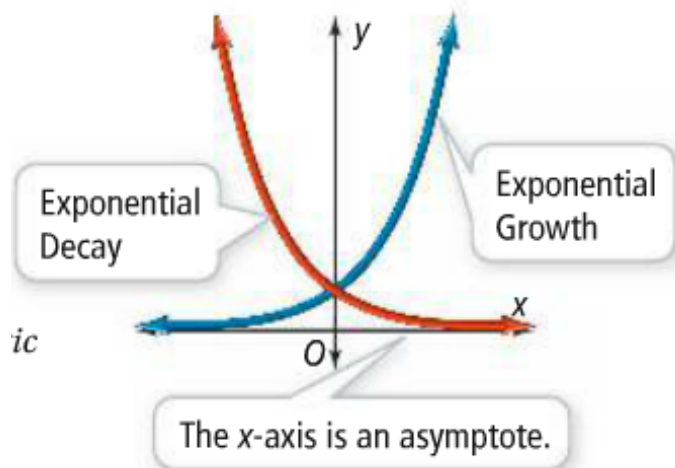
x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	



4. What generalization can you make about the domain, range, and y-intercept of these functions?

### Two types of exponential behavior

1. **Exponential Growth** – as the value of  $x$  increases, the value of  $y$  increases
2. **Exponential Decay** – as the value of  $x$  increases, the value of  $y$  decreases, approaching zero



**Asymptote:** is a line that a graph approaches as  $x$  or  $y$  increase in absolute value

## Concept Summary Exponential Functions

For the function  $y = ab^x$ ,

- if  $a > 0$  and  $b > 1$ , the function represents exponential growth.
- if  $a > 0$  and  $0 < b < 1$ , the function represents exponential decay.

In either case, the  $y$ -intercept is  $(0, a)$ , the domain is all real numbers, the asymptote is  $y = 0$ , and the range is  $y > 0$ .

You Try:

Identify each function or situation as an example of exponential growth or decay. What is the  $y$ -intercept?

5.  $y = 12(0.95)^x$

6.  $y = 0.25(2)^x$

7.  $y = 3(4^x)$

8.  $y = 0.7^x$

For exponential growth  $y = ab^x$ , with  $b > 1$ , the value of  $b$  is the **growth factor**.

*A quantity that exhibits exponential growth increases by a constant percentage each time period. The percentage increase  $r$ , written as decimal, is the rate of increase or growth rate. For exponential growth,  $b = 1 + r$ .*

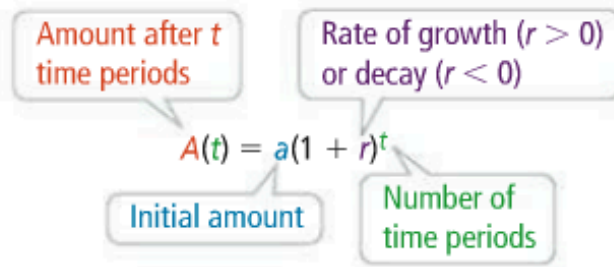
For exponential decay  $y = ab^x$ , with  $0 < b < 1$ , the value of  $b$  is the **decay factor**.

*The quantity decreases by a constant percentage each time period. The percentage decrease,  $r$ , is the rate of decay. Usually the rate of decay is expressed as a negative quantity, so  $b = 1 - r$ .*

Take note

## Key Concept Exponential Growth and Decay

You can model exponential growth or decay with this function.



For growth or decay to be exponential, a quantity changes by a fixed percentage each time period.

9. In 1990, the tuition at a private college was \$15,000. During the next 9 years, tuition increased by about 7.2% each year.
- Write a model giving the cost,  $C$ , of tuition at the college  $t$  years after 1990.
  - Estimate the tuition in 2018.
10. In January 1993, there were about 1.313 million Internet hosts. During the next five years, the number of hosts increased by about 100% per year.
- Write a model giving the number of hosts (in millions)  $t$  years after 1993.
  - About how many hosts were there in 1996?

11. A tool and die business purchases a piece of equipment for \$250,000. The value of the equipment depreciates at a rate of 15% each year.

a. Write an exponential model for the value of the equipment.

b. What is the value of the equipment after 5 years?

12. You drink a beverage with 120 milligrams of caffeine. Each hour the amount of caffeine in your system decreases by about 12%. How much caffeine will be in your system in 3 hours?

**Compound interest:** When a bank pays interest on both the principal and the interest on money already earned.

**Formula:**  $A = P \left( 1 + \frac{r}{n} \right)^{nt}$

A = the balance (annual interest earned)

P = principal ( initial deposit)

r = annual interest rate (as a decimal)

n = number of times interest is compounded per year

Yearly(annually) – 1

Semi-annually – 2

Quarterly – 4

Monthly – 12

Daily – 365 days

t = time in years

You Try:

13. You invested \$1000 in a savings account at the end of 6<sup>th</sup> grade. The account pays 5% annual interest. How much money will be in the account after six years?
14. Suppose you invest \$500 in a savings account that pays 3.5% annual interest. How much will be in the account after 5 years?
15. You buy a savings bond for \$25 that pays a yearly interest rate of 4.2%. What will the savings bond be worth after 15 years?
16. Suppose you invest \$1000 in a savings account that pays 5% annual interest. If you make no additional deposits or withdrawals, how many years will it take for the account to grow to at least \$1500?

17. Suppose you invest \$500 in a savings account that pays 3.5% annual interest. When will the account contain at least \$650?

18. You open a savings account that pays 4.5% annual interest. If your initial investment is \$300 and you make no additional deposits or withdrawals, how many years will it take for the account to grow to at least \$500?

19. You put \$1000 into a college savings account for 4 years. The account pays 5% interest annually. Estimate the amount of money in the savings account after 4 years.

20. Suppose that when your friend was born, your friend's parents deposited \$2000 in an account paying 4.5% interest compounded quarterly. What will the account balance be after 18 years?

a) Write a model giving the amount of money,  $A$ ,  $t$  years after your friend was born.

b) What will be the account balance after 18 year?



21. You deposit \$1000 in an account that earns 2.5% annual interest. Find the account balance after 3 years if this interest is compounded monthly.
22. You deposit \$400 in an account that pays 2% annual interest compounded quarterly. How much will be in the account in three years?
23. You deposit \$975 in an account that pays 5.5% annual interest compounded continuously. What is your balance after 6 years?

Exponential functions are often discrete. The graph consists of individual points corresponding to each  $t=1, 2, 3$  and so on. It is not continuous because at times it will never be 1 set amount.

To model a discrete situation using an exponential function of the form  $y = ab^x$ , you need to find the growth or decay factor  $b$ .

1. you must know the  $y$ -values for two consecutive  $x$ -values
2. you can find the rate of change  $r$
3. then find  $b$  using  $r = \frac{(y_2 - y_1)}{y_1}$  and  $b = 1 + r$

24. In 2006, the world population of the Iberian lynx was 150. In 2007, it was 120. Assume this trend continues.

a. Write an exponential model for the population.

b. What is the population in 2017?

25. Before a basketball game, a referee noticed that the ball seem under-inflated. She dropped it from 6 feet and measured the first bounce as 36 inches and the second bounce as 18 inches.

a. Write an exponential function to model the height of the ball.

b. How high was the ball on its fifth bounce?

26. The initial value of a car is \$30,000. After one year, the value of the car is \$20,000. Estimate the value of the car after 5 years.

Closure: How can you determine the growth rate or decay rate for an exponential function given two consecutive y-values?

---

---

---

---

Assignment: section 7.1 # 10,14,18,19,20,22,26,27,28,29,31

(11 problems)