

Section 10-5 Hyperbolas

Learning Goal: To understand how to write the equation of a hyperbola; to find and use the foci of a hyperbola and to graph a hyperbola.

Essential Questions:

What is the intersection of a cone and a plane parallel to a line along the side of a cone?

What is the difference between the algebraic representations of ellipses and hyperbolas?

Does the graph of a conic section ever intersect its focus or foci?

Which conic sections are symmetric? How can you identify their axes of symmetry?

Compare the standard-form equations of the conic sections. How can you identify each type of conic from the equation?

Warm Up:

1. Which function best represents the graph?

a) $y = \frac{1}{x-1}$

b) $y = \frac{x}{x-1}$

c) $y = \frac{1}{x+1}$

d) $y = \frac{x}{x+1}$

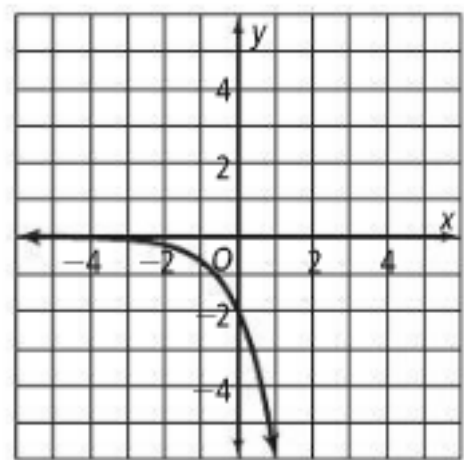
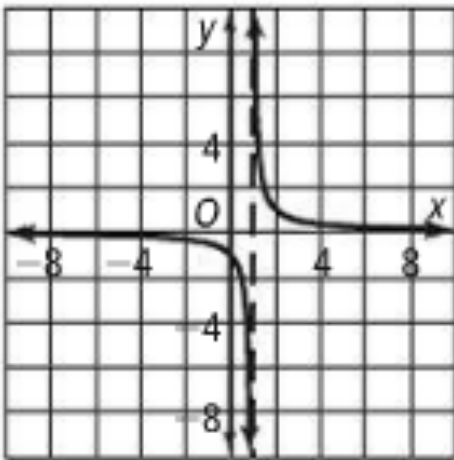
2. Which function best represents the graph?

a) $f(x) = 2 \cdot 3^{-x}$

b) $f(x) = -2 \cdot 3^x$

c) $f(x) = 2 \cdot 3^x$

d) $f(x) = -2 \cdot 3^{-x}$



3. What is the equation of a parabola with the following characteristics?

Axis of symmetry: $x = -3$

Range: all real numbers less than or equal to 4

4. Solve:

$$\frac{3}{2x+10} + \frac{5}{4} = \frac{7}{x+5}$$

a) $y = -(x-4)^2 - 3$

b) $y = -(x+3)^2 + 4$

a) $-\frac{50}{11}$

b) $-\frac{9}{5}$

c) $y = (x-4)^2 - 3$

d) $y = (x+3)^2 + 4$

c) $-\frac{34}{10}$

d) $-\frac{3}{5}$

Vocabulary:

Hyperbola - a set of points P in a plane such that the absolute value of the difference between the distances from P to two fixed points F_1 and F_2 is a constant k

Focus of an hyperbola - (plural: foci) one of the two fixed points

Vertex - the turning point of each branch of the hyperbola

Transverse axis - the segment connecting the two vertices

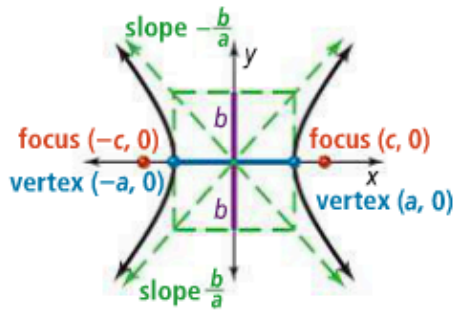
Axis of symmetry - the transverse axis lies on this axis

Center of the hyperbola - the midpoint between the two vertices, which is also the midpoint between the two foci.

Take note

Key Concept Properties of Hyperbolas with Center (0, 0)

Horizontal Hyperbola



Equation: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

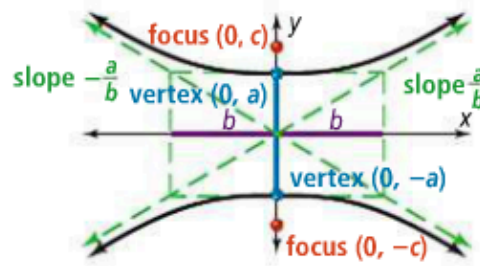
Transverse axis: Horizontal

Vertices: $(\pm a, 0)$

Foci: $(\pm c, 0)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{b}{a}x$

Vertical Hyperbola



Equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Transverse axis: Vertical

Vertices: $(0, \pm a)$

Foci: $(0, \pm c)$, where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{a}{b}x$

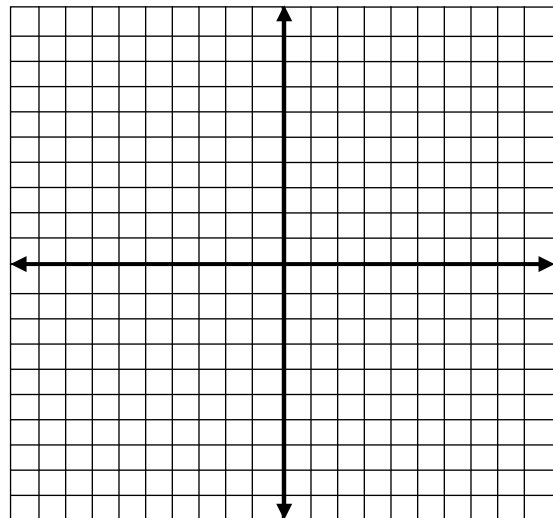
Standard form of the equation of a hyperbola with center at (0, 0) is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

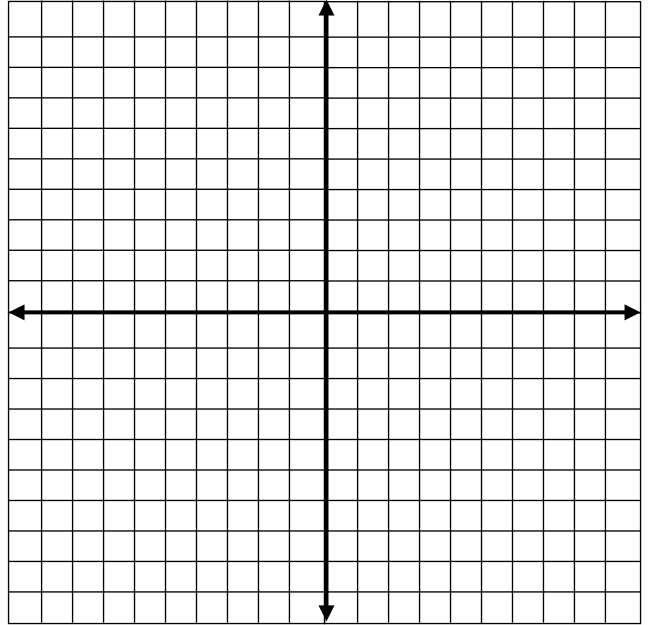
$$c^2 = a^2 + b^2$$

Writing and graphing the equation of a hyperbola.

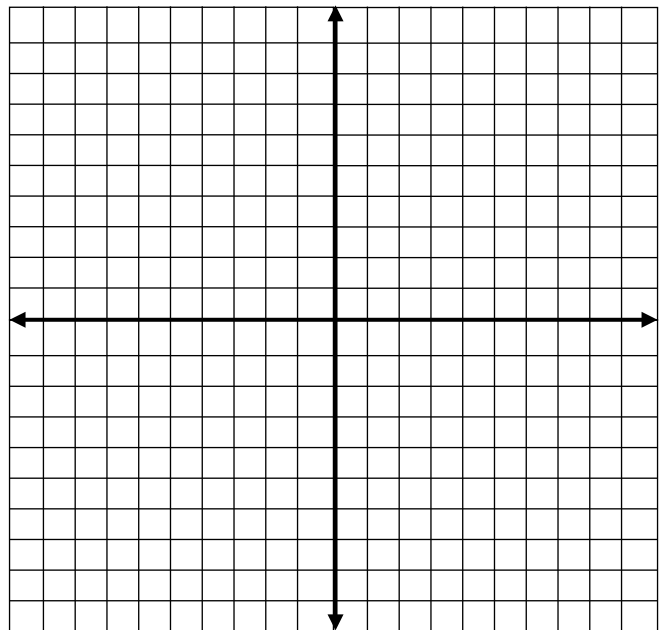
1. What is an equation in standard form of a hyperbola centered at the origin has vertices $(\pm 4, 0)$ and one focus $(5,0)$. What is the sketch of the hyperbola.



2. What is an equation in standard form of a hyperbola centered at the origin has vertices $(0, \pm 4)$ and one foci $(0, \pm 5)$. What is the sketch of the hyperbola.



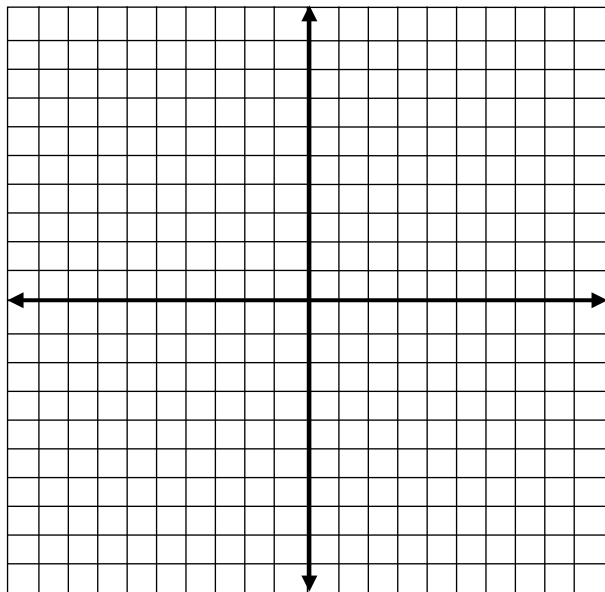
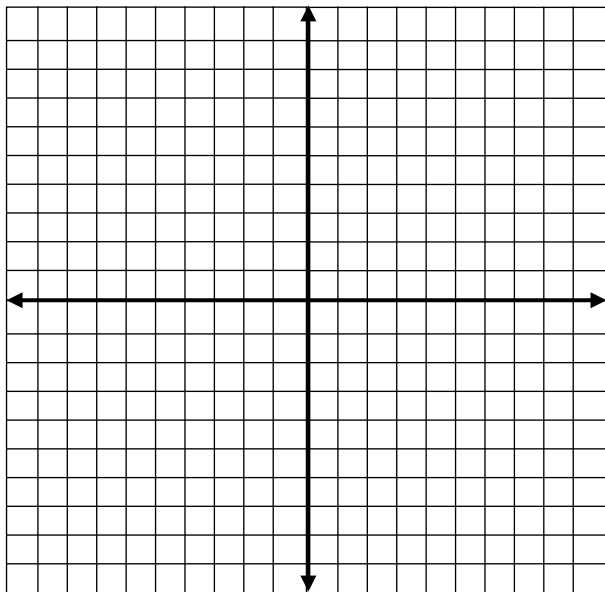
3. What is an equation in standard form of a hyperbola centered at $(0,0)$ has vertices $(\pm 2,0)$ and one focus $(3,0)$. What is the sketch of the hyperbola.



Graph each hyperbola. Find the coordinates and identify the foci

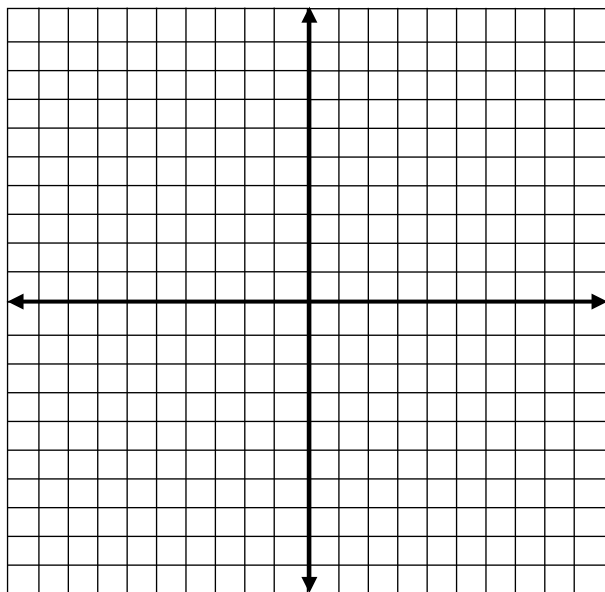
4. $9y^2 - 7x^2 = 63$

5. $\frac{x^2}{64} - \frac{y^2}{17} = 1$



Analyzing a hyperbola from its equation

6. What are the vertices, foci, and asymptotes of the hyperbola with equation $4y^2 - x^2 = 16$? Sketch the graph.



Closure: What parts of a hyperbola must be included on its graph

Assignment: section 10.5 # 12,13,14,15,20,21,25,26 (8 problems)